Chapter 4 Equivalence Calculations under Inflation

4.1)

$$1.1(1+f)^{11} = 3.15$$
$$f = 10.04\%$$
$$100(1+0.1004)^{11} = 286.45$$

4.2) (a)

$$144.5(1+f)^{5} = 170.6$$
$$f = 3.3766\%$$

(b) $170.6(1+0.033766)^{2} = 182.32$

4.3)

$$100(1+0.05)(1+0.08) = 113.40$$
$$100(F/P, \bar{f}, 2) = 113.40$$
$$\bar{f} = 6.4894\%$$

4.4)

$$f_1 = \frac{538{,}400 - 504{,}000}{504{,}000} = 6.825\%$$
$$f_2 = \frac{577{,}000 - 538{,}400}{538{,}400} = 7.169\%$$
$$f_3 = \frac{629{,}500 - 577{,}000}{577{,}000} = 9.099\%$$
$$\bar{f} = \left(\frac{629{,}500}{504{,}000}\right)^{1/3} - 1 = 7.69\%$$

4.5)

$$\text{Given}: f = 7\%$$

$$\begin{aligned} 1(1+0.07)^N &= 2 \\ 1.07^N &= 2 \\ (N)\log 1.07 &= \log 2 \\ N &= \log 2 / \log 1.07 \\ &= 10.24 \text{ years} \end{aligned}$$

Comments: If you use the Rule of 72, you may find

$$\frac{72}{7} = 10.29 \text{ years}$$

which is very close to the actual value.

4.6)

$$\begin{aligned} \$70,000 &= \$55,000(1+\bar{f})^5 \\ (1+\bar{f})^5 &= 1.27273 \\ \bar{f} &= 4.94\% \end{aligned}$$

4.7) Given: $i = 12\%, \bar{f} = 5\%$, 10 annuity payments in actual dollars

$$\begin{aligned} P &= \$8,500(P/A,12\%,10) \\ &= \$48,026.7 \end{aligned}$$

Comments: Since the annuity payments are made in actual dollars, we use the market interest rate to find its equivalent lump sum amount in today's dollars.

4.8) Given: $i = 15\%, \bar{f} = 8\%$, maintenance costs are given in constant dollars,

$$i' = \frac{0.15 - 0.08}{1 + 0.08} = 6.48\%$$

$$\begin{aligned}P &= \$25,000(P/F,6.48\%,1)+\$30,000(P/F,6.28\%,2)\\ &\quad+\ \$32,000(P/F,6.48\%,3)+\$35,000(P/F,6.48\%,4)\\ &\quad+\ \$40,000(P/F,6.48\%,5)\\ &= \$132,894\\ A &= \$132,894(A/P,15\%,5)\\ &= \$39,644.29\end{aligned}$$

4.9) Given: $i = 16\%, \bar{f} = 4\%$

n	Actual dollars	Constant Dollars
0	\$2,500	\$2,500(*P/F*,4%,0) = \$2,500
4	\$4,500	\$4,500(*P/F*,4%,4) = \$3,846.6
5	\$3,500	\$3,500(*P/F*,4%,5) =\$ 2,876.65
7	\$5,500	\$5,500(*P/F*,4%,7) = \$4,179.45

4.10) Given: $P = \$12,000$, $i = 1\%$ per month, $\bar{f} = 0.5\%$ per month

- 20th payment in actual dollars:

  $A_{20} = \$12,000(A/P,1\%,48) = \$315.6$

- 20th payment in constant dollars:

  $A'_{20} = \$315.6(P/F,0.5\%,20) = \$285.65$

4.11) Given: $i = 13\%, \bar{f} = 7\%$

(a) Constant-dollar analysis: we need to find the inflation-free interest rate.

$$i' = \frac{i - \bar{f}}{1 + \bar{f}} = 5.607\%$$

Then, find the equivalent present worth of this geometric series at i'.

$$\begin{aligned}P &= \$15,000(P/A_1,8\%,5.607\%,4)\\ &= \$58,774.83\end{aligned}$$

(b) Actual-dollar analysis

Period	Net Cash Flow in Constant $	Conversion factor	Net Cash Flow in Actual $
1	$15,000	$(1+0.07)^1$	$16,050
2	$16,200	$(1+0.07)^2$	$18,547.4
3	$17,496	$(1+0.07)^3$	$21,433.35
4	$18,895.68	$(1+0.07)^4$	$24,768.38

$$\begin{aligned} P &= \$16,050(P/F,13\%,1)+\$18,547.4(P/F,13\%,2) \\ &\quad +\$21,433.35(P/F,13\%,3)+\$24,768.38(P/F,13\%,4) \\ &= \$58,774.6 \end{aligned}$$

Comments: As an alternative way of finding the equivalent cash flows in actual dollars, we may use the compound growth rate (geometric growth and inflation):

$$\begin{aligned} g &= (1+0.08)(1+0.07)-1 \\ &= 15.56\% \\ P &= \$15,000(1.07)(P/A_1,15.56\%,13\%,4) \\ &= \$58,774.16 \end{aligned}$$

4.12) Given: $i = 9\%, \bar{f} = 3.8\%$, we find the inflation-free interest rate as follows:

$$i' = (0.09-0.038)/(1+0.038) = 5.01\%$$

First compute the equivalent present worth of the constant dollar series at i':

$$\begin{aligned} P &= \$1,000(P/A,5.01\%,3) \\ &= \$2,722.74 \end{aligned}$$

Then, we compute the equivalent equal annual payment in actual dollars using *i*:

$$\begin{aligned} A &= \$2,722.74(A/P,9\%,3) \\ &= \$1,075.63 \end{aligned}$$

4.13) Given: $i = 12\%, \bar{f} = 6\%$, bond interest rate = 9% compounded semiannually, face value = $1,000

- The 16th interest payment in actual dollars:

$$I_{16} = \$1,000(0.045) = \$45$$

- The 16th interest payment (8th year) in constant dollars:

$$I'_{16} = \$45(P/F, 6\%, 8) = \$28.23$$

4.14) Given: $i' = 4\%, \bar{f} = 5\%$

$$i = 0.04 + 0.05 + (0.04)(0.05) = 0.092$$

$$\begin{aligned}&\$30{,}000(1+\bar{f})^5(P/F, 9.2\%, 5)\\&= \$30{,}000(1+0.05)^5(0.644)\\&= \$24{,}657.8\end{aligned}$$

4.15) Given: $i = 1\%$ per month, $\bar{f} = 0.5\%$ per month, P = \$20,000, N = 60 months

$$\begin{aligned}i' &= \frac{0.01 - 0.005}{1 + 0.005}\\&= 0.4975\%\\A' &= \$20{,}000(A/P, 0.4975\%, 60)\\&= \$386.38\end{aligned}$$

4.16) Given: $i' = 6\%, \bar{f} = 5\%, N = 5$ years, $A = \$1.5$ million in constant dollars

- Market interest rate:

$$i = 0.06 + 0.05 + (0.06)(0.05) = 11.3\%$$

- Actual dollar analysis:

Period	Net Cash Flow in Constant \$	Net Cash Flow in Actual \$	Equivalent Present Worth
1	\$1,500,000	\$1,575,000	\$1,415,094
2	\$1,500,000	\$1,653,750	\$1,334,995
3	\$1,500,000	\$1,736,438	\$1,259,429
4	\$1,500,000	\$1,823,259	\$1,188,140
5	\$1,500,000	\$1,914,422	\$1,120,887
		Total	\$6,318,545

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$$\begin{aligned} P &= \$1,575,000(P/F,11.3\%,1) \\ &\quad + \cdots + \$1,914,422(P/F,11.3\%,5) \\ &= \$6,318,545 \end{aligned}$$

4.17) Given: $i = 0.75\%$ per month, $\overline{f} = 0.5\%$ per month, $P = \$5,000$, $N = 24$ months, down payment = $1,000

(a) Inflation-free interest rate:

$$i' = \frac{0.0075 - 0.005}{1 + 0.005} = 0.2488\% \text{ per month}$$

(b) Equal monthly payment in constant dollars:

$$\begin{aligned} A' &= \$5,000(A/P,0.2488\%,24) \\ &= \$214.87 \end{aligned}$$

4.18) Given: $i = 6\%$ compounded monthly, $\overline{f} = 5\%$ compounded annually, number of months to deposit = 240 months, number of annual withdrawals = 15, first withdrawal = 6 months after retirement

- Effective inflation rate per half-year: Since the first withdrawal is made after 6 months from retirement, it is necessary to calculate the effective inflation rate per half-year.

$$\overline{f} = \left(1 + \frac{0.05}{2 \cdot (1/2)}\right)^{1/2} - 1 = 2.4695\% \text{ per half-year}$$

- Annual withdrawals in actual dollars: On a semiannual basis, the first withdrawal will be made after 41 semiannual periods. Then, we can calculate the equivalent amount of this first withdrawal in actual dollars as follows:

$$A_{41} = \$40,000(F/P,2.4695\%,41) = \$108,752$$

The second withdrawal will be made after 43 semiannual periods. The equivalent amount of this second withdrawal in actual dollars is

$$A_{43} = \$40,000(F/P,2.4695\%,43) = \$114,192$$

The remaining withdrawals in actual dollars are

$$
\begin{aligned}
A_{45} &= \$40,000(F\,/\,P, 2.4695\%, 45) = \$119,990 \\
A_{47} &= \$40,000(F\,/\,P, 2.4695\%, 47) = \$125,895 \\
A_{49} &= \$40,000(F\,/\,P, 2.4695\%, 49) = \$132,189 \\
A_{51} &= \$40,000(F\,/\,P, 2.4695\%, 51) = \$138,799 \\
A_{53} &= \$40,000(F\,/\,P, 2.4695\%, 53) = \$145,739 \\
A_{55} &= \$40,000(F\,/\,P, 2.4695\%, 55) = \$153,026 \\
A_{57} &= \$40,000(F\,/\,P, 2.4695\%, 57) = \$160,677 \\
A_{59} &= \$40,000(F\,/\,P, 2.4695\%, 59) = \$168,711 \\
A_{61} &= \$40,000(F\,/\,P, 2.4695\%, 61) = \$177,146 \\
A_{63} &= \$40,000(F\,/\,P, 2.4695\%, 63) = \$186,003 \\
A_{65} &= \$40,000(F\,/\,P, 2.4695\%, 65) = \$195,304 \\
A_{67} &= \$40,000(F\,/\,P, 2.4695\%, 67) = \$205,069 \\
A_{69} &= \$40,000(F\,/\,P, 2.4695\%, 69) = \$215,322
\end{aligned}
$$

• Equivalence calculation: To find the required equal monthly deposit amount (A), we establish the following equivalence relationship:

$$i_a = \left(1 + \frac{0.06}{1 \cdot (12)}\right)^{12} - 1 = 6.168\% \text{ per year}$$

$$
\begin{aligned}
A(F\,/\,A, 0.5\%, 240)(F\,/\,P, 0.5\%, 6) &= \$108,752 \\
&\quad + \$114,192(P\,/\,F, 6.168\%, 1) \\
&\quad + \$119,900(P\,/\,F, 6.168\%, 2) \\
&\quad \vdots \\
&\quad + \$215,322(P\,/\,F, 6.168\%, 14) \\
&= \$1,511,533.1 \\
A &= 3,174.91 \text{ per month}
\end{aligned}
$$

4.19) Given: $i = 2\%$ per quarter, $\overline{f} = 6\%$ per year

(a)

• Actual dollar analysis:

$$A(F/A,2\%,160) = \$600,000(F/P,6\%,40)$$
$$= \$6,171,431$$
$$A = \$5,420.69$$

(b)

- Effective annual interest rate:

$$i_a = (1+0.08/4)^4 - 1 = 8.243\%$$

- Equivalent value of \$600,000 in actual dollars at the end of 63rd birthday:

$$\$600,000(F/P,6\%,40) = \$6,171,431$$

- Conversion of gradient series to equivalent uniform series:

$$A = G(A/G,8.243\%,40)$$
$$= \$1,000(10.3745)$$
$$= \$10,374$$

- Amount of the first deposit (A_1):

$$(A_1 + \$10,374)(F/A,8.243\%,40) = \$6,171,431$$

$$A_1 = \$11,968.6$$

4.20)

$$i = i' + \bar{f} + i'\bar{f} = 0.06 + 0.05 + 0.06(0.05) = 0.113$$

$$A(F/A,11.3\%,8) = \left[40,000(P/A,11.3\%,4) + 1,000(P/G,11.3\%,4)\right](F/P,11.3\%,1)$$
$$11.9897A = \$141,930.65$$

$$A = \$11,837.72$$

4.21) Given: $i = 8\%$ per year, $\bar{f} = 6\%$ per year

(a) Freshman-year expense in actual dollars:

$$\$40,000(F/P,6\%,10) = \$71,632$$

(b) Equivalent single-sum amount at $n = 0$

$$\begin{aligned} i' &= \frac{i - \bar{f}}{1 + \bar{f}} \\ &= (0.08 - 0.06)/(1 + 0.06) \\ &= 0.01887 \\ P &= [\$40,000(P/A, 1.887\%, 3) \\ &\quad + \$40,000](P/F, 1.887\%, 10) \\ &= \$129,076.84 \end{aligned}$$

(c) Required annual deposit in actual dollars:

$$A = \$129,076.84(A/P, 8\%, 10) = \$19,236.2$$

4.22)

(a) The average annual general inflation rate:

$$\begin{aligned} (1 + 0.065)(1 + 0.077)(1 + 0.081) &= 1.2399 \\ (1 + \bar{f})^3 &= 1.2399 \\ \bar{f} &= 7.4308\% \end{aligned}$$

(b) Constant dollars:

n	Actual dollars	Constant dollars
0	-$45,000	-$45,000
1	$26,000	$26,000(0.9390) = $24,414
2	$26,000	$26,000(0.8718) = $22,667
3	$26,000	$26,000(0.8065) = $20,969

Conversion factors:

$$\begin{aligned} (P/F, 6.5\%, 1) &= 0.9390 \\ (P/F, 7.7\%, 1)(P/F, 6.5\%, 1) &= 0.8718 \\ (P/F, 8.1\%, 1)(P/F, 7.7\%, 1)(P/F, 6.5\%, 1) &= 0.8065 \end{aligned}$$

(c)

$$\begin{aligned} P &= -\$45,000 + \$24,414(P/F, 5\%, 1) \\ &\quad + \$22,667(P/F, 5\%, 2) + \$20,969(P/F, 5\%, 3) \\ &= \$16,923.88 \end{aligned}$$

4.23) Let i be the effective interest rate per month. Then,

$$(1+i)^{12} - 1 = 0.0677$$
$$(1+i)^{12} = 1 + 0.0677$$
$$i = (1+0.0677)^{1/12} - 1$$
$$= 0.5474\%$$

$$P = \$10{,}000 + \$100(P/A, 0.5474\%, 480) = \$26{,}938.67$$

4.24) To find the long-term average tuition inflation rate, we first need to find out what the tuition would be in 2005 using year 1978 as a base period. As shown in the following table, it will cost almost eight (7.9296) times higher than year 1978.

Year (*n*)	**Tuition CPI (%)**	**Compound Factor**
2005	7.46	7.9296
2004	9.46	7.3791
2003	8.4	6.7414
2002	6.82	6.2190
2001	5.09	5.8219
2000	4.14	5.5399
1999	3.98	5.3197
1998	4.22	5.1161
1997	5.11	4.9089
1996	5.66	4.6703
1995	6	4.4201
1994	6.98	4.1699
1993	9.37	3.8978
1992	10.74	3.5639
1991	10.17	3.2183
1990	8.09	2.9212
1989	7.93	2.7025
1988	7.6	2.5040
1987	7.56	2.3271
1986	8.09	2.1636
1985	9.1	2.0016
1984	10.23	1.8347
1983	10.41	1.6644
1982	13.44	1.5075
1981	12.43	1.3289
1980	9.43	1.1820
1979	8.01	1.0801
1978	Base	1.0000

That is,

$$7.9296 = 1(1+f)^{27}$$
$$f = 7.97\%$$

If a baby born in 2005 goes to college at the age of 18, the expected college tuition each school year is as follows:

$$\begin{aligned} \text{Freshman: } & \$18,000(F/P, 7.97\%, 18) = \$71,570 \\ \text{Sophomore: } & \$18,000(F/P, 7.97\%, 19) = \$77,274 \\ \text{Junior: } & \$18,000(F/P, 7.97\%, 20) = \$83,432 \\ \text{Senior: } & \$18,000(F/P, 7.97\%, 21) = \$91,600 \end{aligned}$$

There are many ways to meet the future college expenses. One of the options is to consider opening a mutual fund account and make regular contribution, say monthly, until the child reaches 18. Let's assume that the mutual fund would grow at an 8% annual compound return. Then, we may be able to estimate the required monthly contribution (C) as follows:

$$\begin{aligned} V_{18} &= \$71,570 + \$77,274(P/F, 8\%, 1) \\ &\quad + \$83,432(P/F, 8\%, 2) + \$91,600(P/F, 8\%, 3) \\ &= \$287,365 \end{aligned}$$

$$C(F/A, \frac{8\%}{12}, 216) = \$287,365$$
$$C = \frac{\$287,365}{480.0861}$$
$$= \$598.57$$

Since there is no way of knowing that the mutual fund will generate an 8% return over 18 years, it would be a good idea to increase the monthly contributions regularly to meet potential shortfalls in case the fund does not perform as expected.

4.25)

(a) Real after-tax yield on bond investment:

- Nontaxable municipal bond:

$$i'_{\text{municipal}} = \frac{0.09 - 0.03}{1 + 0.03} = 5.825\%$$

- Taxable corporate bond:

$$i'_{\text{corporate}} = \frac{0.12(1-0.3)-0.03}{1+0.03} = 5.243\%$$

The municipal bond provides a greater return on investment.

(b)

Given : $i = 6\%$, and $\bar{f} = 3\%$,

$$i'_{\text{savings}} = 2.91\%$$

Since $i'_{\text{municipal}}$ >2.91% and $i'_{\text{corporate}}$ >2.91%, both bond investments are better than the savings account. Now to compare two mutually exclusive bond investment alternatives, we need to perform an incremental analysis.

After-tax Cash Flow			
n	Municipal	Corporate	Incremental
0	-$10,000	-$10,000	0
1	$900	$840	-$60
2	$900	$840	-$60
3	$900	$840	-$60
4	$900	$840	-$60
5	$900	$840	-$60

We cannot find the rate of return on incremental investment, as returns from municipal bond dominate those from corporate bond in every year. Municipal bond is a clear choice for any value of MARR.

4.26)

Two common approaches may be used: either (1) constant dollar analysis or (2) actual dollar analysis. In this case, it may be easier to use the constant dollar analysis, as we don't need to project the future price increase of the subscription, assuming that the price of magazine will follow the general inflation rate. Then, we need to determine which interest rate to use in evaluating the three different subscription options. Assuming that the decision-maker's desired inflation-free interest rate (i') or real earnings from his or her personal investment is around 2%, we can determine the total subscription cost for life-time (say, more than 50 years) as follows:

$$P_{\text{1-year subscription}} = \$39 + \frac{\$39}{0.02} = \$1,989$$
$$P_{\text{2-year subscription}} = \$72 + \frac{\$72}{0.0404} = \$1,854$$
$$P_{\text{3-year subscription}} = \$103 + \frac{\$103}{0.06121} = \$1,786$$

In this case, the 3-year subscription option appears to be a better choice. Note that 4.04% represents the effective interest rate for 2 years and 6.121% does for 3 years. The view taken in this calculation is that if the general inflation rate were running at 3%, the decision-maker would earn around 5% (=3% + 2%) market interest rate. Certainly the choice will change depending upon the individual decision-maker's true earrings requirement.

If we take a finite planning horizon, say 6-year, the subscription cost for each option would be as follows:

$$P_{\text{1-year subscription}} = \$39 + \$39(P/A, 2\%, 5) = \$222.82$$
$$P_{\text{2-year subscription}} = \$72 + \$72(P/F, 2\%, 2) + \$72(P/F, 2\%, 4) = \$207.72$$
$$P_{\text{3-year subscription}} = \$103 + \$102(P/F, 2\%, 3) = \$200.05$$

It still appears that the 3-year subscription is a better choice.

4.27) Four-year Full Benefits Plan for The University of Michigan at Ann Arbor:

- Determine the average inflation rate for tuition and fees:

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$$\$7,560 = \$3,191(1+f)^{13}$$
$$f = 6.86\%$$

- Compute the anticipated annual tuitions and fees from the perspective of a parent with a newborn in 2001, assuming that the future tuition and fees will increase at the annual rate of 6.86%:

Birthday	Expected Tuition and Fees
18 Freshman	\$7,560(*F/P*,6.86%,18) = \$24,957
19 Sophomore	\$7,560(*F/P*,6.86%,19) = \$26,669
20 Junior	\$7,560(*F/P*,6.86%,20) = \$28,498
21 Senior	\$7,560(*F/P*,6.86%,21) = \$30,453

- Determine the amount of accumulated savings versus the required college savings in actual dollars:

Sample calculations at i = 8%:

$$F_{\text{deposit}} = \$24,252(1+0.08)^{18} = \$96,911$$
$$\begin{aligned} P_{\text{required savings}} &= \$24,957 + \$26,669(P/F,8\%,1) \\ &\quad + \$28,498(P/F,8\%,2) + \$30,453(P/F,8\%,3) \\ &= \$98,202 \end{aligned}$$

Savings Rate (Market rate)	Accumulated Savings at age of 18 years	Required College Savings at age of 18 years
5%	\$58,365	\$102,453
6%	\$69,223	\$100,991
7%	\$81,970	\$99,975
8%	\$96,911	\$98,202
10%	\$134,839	\$95,578

The breakeven interest rate is about 8.07%. In other words, if you cannot invest your money at higher than 8.07%, you are better off with the State's Full Benefits Plan.